Exercise 1

Find the general solution for each of the following first order ODEs:

$$u' + u = e^{-x}, \ x > 0$$

Solution

This is an inhomogeneous first order linear ODE, so we can multiply both sides by the integrating factor,

$$I(x) = e^{\int dx} = e^x,$$

to solve it. The equation becomes

$$e^x u' + e^x u = 1.$$

Observe that the left side can be written as $(e^x u)'$ by the product rule.

$$\frac{d}{dx}(e^x u) = 1$$

Now integrate both sides with respect to x.

$$e^x u = x + C$$

Therefore,

$$u(x) = e^{-x}(x + C), \ x > 0.$$